Accurate Frequency Domain Modelling of Coaxially Driven Axisymmetric Microwave Structures

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Abstract— **This letter presents a simple, yet accurate, finite-element frequency domain method of analysis applicable to coaxially driven, axisymmetric microwave structures. The method combines known techniques of modelling coaxially driven, non-radiating structures with those used to model radiating, non-coaxially driven microwave structures. As a result, the method is capable of modelling both types of structures. As an example of the various possible applications of the method, a monopole antenna realized with a ring resonator was analyzed and the results are reported. The results agree very well with the reported measured results for that antenna.**

I. Introduction

FREQUENCY domain analysis of coaxial structures has been attempted by many researchers [1]–[3]. Their analysis, however, only deals with cases where the input and output ports of the structures are adapted for coaxial line connection. In other words, it would not apply to general antenna structures radiating in open regions with only one coaxial input port. On the other hand, the literature is full of frequency domain studies of radiating structures, e.g., [4], but, unfortunately, the feeds are sometimes only approximated as delta gap voltage or field functions or TEM frill generators, so they do not accurately model the practical coaxial feeding process. This letter sets out to overcome those shortcomings in a very simple, and yet accurate, way.

II. The Method

The method used implements the frequency-domain finite-element method (FEM) in a manner similar to that used in [4]. For an axisymmetric structure and a TEM excitation, the Maxwell's curl equations in cylindrical coordinates with electric field, E, eliminated, can be reduced to a scalar form as

$$-\frac{\partial}{\partial z}\left(\frac{\partial H_\phi}{\partial z}\right)-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rH_\phi)\right)=(-j\omega\mu H_\phi)(\sigma+j\omega\varepsilon) \tag{1}$$

Through thorough investigation of the method introduced in [4], it was found that solving for the azimuthal component of the magnetic field, H_ϕ, simplifies a lot the working out of the associated integrals rather than solving for rH_ϕ, as is the case in [4]. This technique further makes it possible to go around the singularity at $r=0$, which was unavoidable in the previous formulation. It was further found that preservation of symmetry claimed under rH_ϕ formulation, is not important when sparse matrix solving routines are used that do not necessarily care about the symmetry of the system. More advantages of the general H-field formulation are discussed in [5], [6].

The Galerkin procedure was used with a trial solution for H_ϕ for the kth element consisting of n nodes, given as

$$H_\phi^{(k)}\approx\sum_{j=1}^{n}u_jN_j^{(k)}(r,z)=U^{(k)}(r,z) \tag{2}$$

where u are unknown complex constants, N are real-valued FEM interpolatory functions, and U are the approximate values of H_ϕ. The undetermined coefficients u_j would reflect the values of U when solved.

Following the procedure introduced in [4], the subsystem of n algebraic equations corresponding to a generic element k is obtained as

$$\begin{aligned}\int\int_{(k)}&\left[Y\frac{\partial N_i}{\partial r}\frac{\partial U^{(k)}}{\partial r}+Y\frac{\partial N_i}{\partial z}\frac{\partial U^{(k)}}{\partial z}+\frac{Y}{r}\frac{\partial N_i}{\partial r}U^{(k)}+ZN_iU^{(k)}\right]drdz\\&=\oint_{(k)}\left[\frac{Y}{r}\frac{\partial(rU^{(k)})}{\partial r}\cos\alpha^{(k)}+Y\frac{\partial U^{(k)}}{\partial z}\cos\beta^{(k)}\right]N_idl\qquad i=1,2,\ldots,n\end{aligned} \tag{3}$$

or after some manipulation as discussed in [4],

$$\begin{aligned}\sum_{j=1}^{n}u_j\int\int_{(k)}&\left[Y\frac{\partial N_i}{\partial r}\frac{\partial N_j}{\partial r}+Y\frac{\partial N_i}{\partial z}\frac{\partial N_j}{\partial z}+\frac{Y}{r}\frac{\partial N_i}{\partial r}N_j+ZN_iN_j\right]drdz\\&=\oint_{(k)}E_{\tan(\mathrm{CCW})}^{(k)}N_idl\qquad i=1,2,\ldots,n\end{aligned} \tag{4}$$

where $Y=(\sigma+j\omega)^{-1}, Z=j\omega\mu$, and $E_{\tan(\mathrm{CCW})}$ is the tangential, counterclockwise, component of the electric field on element sides. Such subsystems are to be assembled for all elements in the problem domain in the usual FEM fashion and solved for the unknown values of u. Sparse matrix solving routines are strongly recommended for both memory and computation efficiencies.

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III. Boundary Conditions

The boundary conditions of the problem are easily introduced into the formulation by specifying the appropriate values of $E_{\tan(\mathrm{CCW})}$ in (4) above. It is known that tangential electric field is zero over all perfect conducting boundaries, such as coaxial line conductors and the perfect ground plane.

Since only half of the problem geometry needs to be solved for axisymmetric configurations, there are cases, such as in antennas, whereby a nonconducting boundary would be required along the axis of symmetry. It is only useful to remember that along such a boundary, H_ϕ is zero and needs not to be computed. In other words, one can safely eliminate the nodes falling on the axis of symmetry before embarking on the solution of the system of algebraic equations.

Treatment of an open boundary, when present, or any matched output termination is done in the same manner as the radiation boundary in [4]. In that case, the tangential electric field on the right-hand side of (4) is written in terms of the unknown magnetic field, using

$$E_{\tan(\mathrm{CCW})} = \eta H_\phi \approx \eta U \approx \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} U \qquad (5)$$

Our investigation has shown that at the feed boundary (i.e., the boundary placed inside the feeding coaxial line close to the driven structure), the technique used in [1] works best when used along with the technique discussed in [4]. In that case, the following analytical expression is used for the input coaxial port:

$$\frac{\partial H_\phi}{\partial z} = -jk(2H_I - H_\phi) \qquad (6)$$

where H_I is the incident magnetic field and $k = \omega\sqrt{(\mu\varepsilon)}$, is the wave number. In terms of the approximate value of the magnetic field, U, (6) can be written as

$$\frac{\partial U}{\partial z} = -jk(2H_I - U) \qquad (7)$$

The boundary conditions at the coaxial input port can be effectively implemented by substituting the derivative of U with respect to z on the right-hand side of (3) using (7). Moreover, the direction cosines of a vector normal to the feed boundary, cosα and cosβ, are respectively specified as 0 and -1 if the boundary is perpendicular to the z-axis and the wave is assumed to propagate in the positive z-direction.

Observe that the above implementation of the boundary conditions at the input port does not assume TEM aperture (junction) fields neither does it fix the excitation as a constant delta voltage or field function. Certainly, the field at any point inside the coaxial line is a function of both incident and reflected fields. That is properly considered in this formulation.

IV. Results

The formulation discussed above has been implemented successfully in analyzing various axisymmetric microwave structures such as cylindrical and conical monopole antennas, coaxial junctions, and dielectric resonators, all driven through coaxial lines. The method can generate various outputs such as transfer characteristics and nodal magnetic field values. The near field values calculated for antenna cases may be transformed to far field by well known near-to-far field transformation algorithms available in the literature.

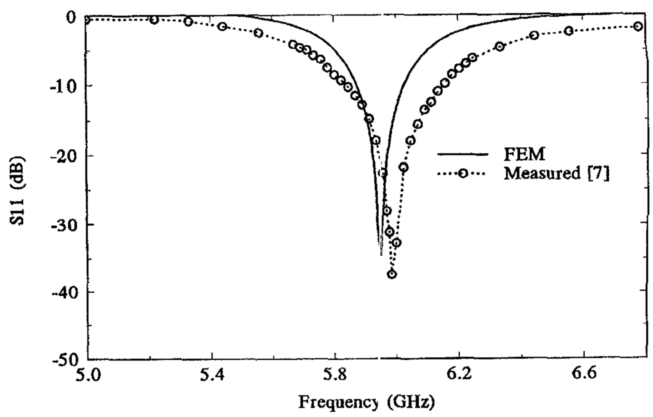

Fig. 1. Measured and numerical results for the input return loss of a cylindrical dielectricring resonator antenna.

In Fig. 1, we present the reflection coefficient (S11) results for the dielectric resonator antenna introduced in [7] and shown in the same paper. The FEM results compare very well with the experimental results reported in [7]. The dielectric resonator antenna modeled is seen to resonate at around 5.95 GHz instead of around 6.0 GHz, found in [7]. The difference is primarily due to the fact that there may be slight differences in the modeled problem dimensions compared to those used in the experiment. For example, the width of the ground plane is only assumed to be of typical value, $t = 1.67$ mm, and the diameter of the ground plane aperture is assumed to be the same as that of the center hole of the cylindrical dielectric resonator ring, i.e., $2b$. These dimensions are critical in fixing the resonant frequency of the resonator as they decide the value of the frequency-dependent capacitance of the aperture. All the other dimensions were taken same as those provided in [7]. That is, $2a = 11.95$ mm, $b = 5.4$ mm, $b/a = 0.17$, probe length, $h = 5.0$ mm and $\epsilon_r = 36.2$.

V. Conclusion

An accurate method of analysis for coaxially driven, axisymmetric microwave structures has been presented. The method accurately models both coaxial feed and open boundaries, hence fixing the shortcomings of all the available methods that only treat well one, but not both, of the above boundaries. The method has been shown to work very well with various coaxially driven axisymmetric structures. It can indeed be used to model monopole antennas of various shapes, dielectric resonators, cavities, filters, coaxial connectors, and virtually all geometries that exhibit axisymmetry.

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